Result of Calibration

It is reasonable to suppose, on the basis of the above estimates, that the calibration of the rotary attenuator by this method should be accurate to ± 0.02 db over a 20-db range, the accuracy being greatest over the first part of the range.

The results obtained are shown in Fig. 5, where the error or deviation in db is plotted for various settings of the rotary attenuator, which is in itself an absolute instrument. For comparison, the same attenuator was calibrated against an IF piston attenuator, the results being shown in the same illustration. The deviations are, in both cases, of the order expected.

Conclusion

The method described above for the absolute calibration of microwave attenuators has been tested experimentally at a wavelength of 3.2 cm. Estimates were made of the padding required to attain an accuracy of the order of ± 0.02 db over a 20-db range. It was shown that the results obtained are accurate to within the design limits of the apparatus, and to within the accuracy of other methods in current use. By further reducing the effects of multiple reflections in the microwave circuit it should be possible to attain even greater accuracy, if required. The method is insensitive to power fluctuations and used comparatively simple and readily available microwave and electronic apparatus.

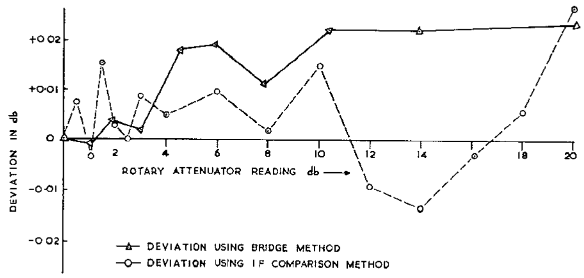

Fig. 5—Deviations of attenuation from rotary attenuator reading using the bridge method and IF comparison method of calibration. The rotary attenuator is an absolute instrument.

Acknowledgment

The author wishes to thank E. A. N. Whitehead for the basic idea, and her other associates of the Microwave Division of Elliott Brothers (London) Limited who helped to put the idea into practice. The comparison with a piston attenuator was carried out at the Radar Research Establishment at Malvern, Worcestershire, and the author would like to acknowledge their cooperation.

Pulse Waveform Degradation Due to Dispersion in Waveguide*

ROBERT S. ELLIOTT†

Summary—**Phase velocity in a waveguide is a nonlinear function of frequency and thus causes dispersion of the spectral components in a pulse waveform. For most practical cases, it is a good assumption to consider the phase constant to be a quadratic function of frequency. An expression can then be derived for the exit waveform shape as a function of guide length, dispersion, and width of the input rectangular pulse. The derived expression is given in terms of tabulated error functions and Fresnel integrals. It is universal in form and applicable to a wide range of practical problems. A family of degraded wave shapes has been computed from this expression and is presented graphically. The results apply for any mode in a straight waveguide of arbitrary but constant cross section.**

Introduction

As THE usable range of microwave frequencies has been pushed higher and higher, a run of waveguide whose physical length is L has assumed an electrical length great enough to affect transmission of pulsed energy, even when loss is ignored. The reason for this lies in the frequency behavior of the phase constant. If $\beta(\omega)$ is the phase constant, then

$$\beta(\omega) = \frac{2\pi}{\lambda_g} = \frac{\sqrt{\omega^2 - \omega_c^2}}{v} \tag{1}$$

in which λ_g is the guide wavelength at the angular frequency ω, and $v = (\mu\epsilon)^{-1/2}$, with μ and ϵ the permeability and permittivity, respectively, of the medium filling the guide. Eq. (1) applies for any mode in a straight section of waveguide of any constant cross section. ω_c is the cutoff angular frequency of the particular mode being considered.

Eq. (1) can be expanded in a Taylor's series about the angular frequency ω_0, giving

$$\beta(\omega) = \beta_0 + \frac{\omega_0}{v^2\beta_0}[\omega - \omega_0] - \frac{1}{2}\frac{\omega_c^2}{v^4\beta_0^3}[\omega - \omega_0]^2 + \frac{1}{2}\frac{\omega_0\omega_c^2}{v^6\beta_0^5}[\omega - \omega_0]^3 - \cdots \tag{2}$$

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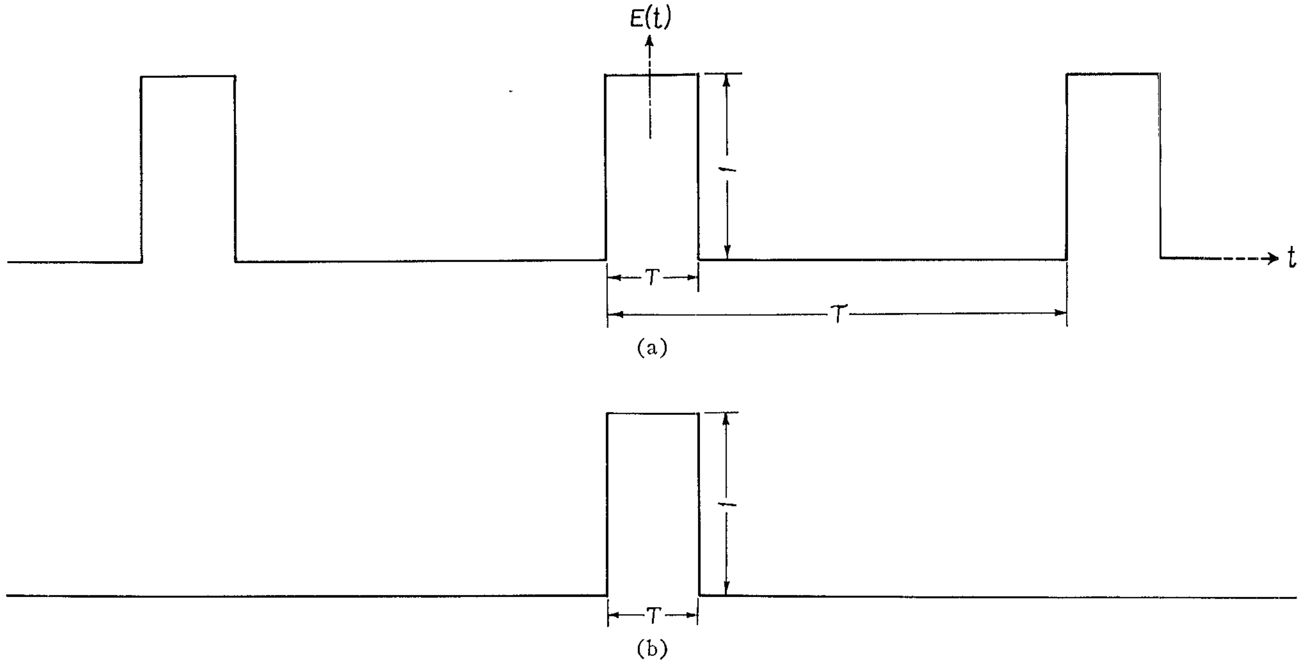

Fig. 1—Rectangular pulse waveforms.

It is apparent from (1) and (2) that $\beta(\omega)$ is not a linear function of ω. Thus, the Fourier components of a pulse traveling down the waveguide are dispersed, and the exit waveform is degraded. The amount of degradation depends on the Fourier composition of the pulse, the length L of the run of waveguide, and the rapidity of convergence of the series (2). This convergence depends on ω_0, ω_c, and the bandwidth $\omega_0-\omega_1\leq\omega\leq\omega_0+\omega_1$ needed to adequately represent the pulse.

This problem of pulse degradation has received both theoretical and experimental attention in the literature, An interesting display of the effect of delay distortion on a rectangular pulse has been provided by the experimental equipment of Beck.[1,2] A series of analytical papers[3-6] has considered the effect of dispersion on a Heaviside unit step function and on a Dirac function, in lossy as well as lossless waveguides. However, in all cases, the analysis was based on the exact relation (1). A precise expression for the exit waveform can be derived in this manner, but, unfortunately, the expression is unwieldy and not given in terms of tabulated functions. Thus, calculated waveshapes which can be compared to Beck's observations have not been readily available.

For most cases of practical interest, an approximation of (1) can be made which yields useful results in a greatly simplified form. This approximation consists of taking only the first three terms of the expansion (2), thus treating $\beta(\omega)$ as a quadratic function of ω. An analysis can be based on this approximation, culminating in an expression for the exit waveform containing only error functions and Fresnel integrals, for both of which adequate tables are available. Sample computations yield a family of graphs of exit waveforms of various degrees of degradation.

Analysis

Let a microwave carrier signal at the angular frequency ω_0 be modulated by a train of rectangular pulses of width T and separation τ. [See Fig. 1(a).] In most practical applications τ/T is so large that the Fourier series spectrum differs negligibly from the Fourier integral spectrum found by considering only one pulse in the train.

Thus, if

$$e(t) = E(t)\cos\omega_0 t \tag{3}$$

is the waveform being injected into a run of waveguide, with $E(t)$ given by Fig. 1(a), it is assumed that a satisfactory model results from saying that (3) is the waveform being injected into the run of waveguide, with $E(t)$ given instead by Fig. 1(b). Then

$$E(t) = \int_{-\infty}^{\infty} g(\omega)e^{j\omega t}d\omega \tag{4}$$

and

$$g(\omega) = \frac{T}{2\pi}\,\frac{\sin\dfrac{\omega T}{2}}{\dfrac{\omega T}{2}}\,. \tag{5}$$

[1] A. C. Beck, "Microwave testing with millimicrosecond pulses," IRE Trans., vol. MTT-2, pp. 93–100; April, 1954.

[2] A. C. Beck, "Measurement techniques for multimode waveguides," IRE Trans., vol. MTT-3, pp. 35–42; April, 1955.

[3] M. Cotte, "Propagation d'une perturbation dans une guide electrique," *Ann. Télécommun.*, vol. 1, pp. 49–52; March–April, 1946.

[4] M. Namiki and K. Horiuchi, "On the transient phenomena in the waveguide," *J. Phys. Soc. Japan*, vol. 7, pp. 190–193; March–April, 1952.

[5] P. Poincelot, "Propagation of a signal along a waveguide," *Ann. Télécommun.*, vol. 9, pp. 315–317; November, 1954.

[6] M. Cotte, "Propagation of a pulse in a waveguide," *Ondé Elec.*, vol. 34, pp. 143–146; February, 1954.

Substitution and symmetry considerations yield

$$e(t) = \frac{T}{2\pi}\int_{-\infty}^{\infty} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \cos(\omega + \omega_0)t \cdot d\omega \qquad (6)$$

as the Fourier equivalent of the input waveform (3).

Referring to the discussion about phase constant in the preceding section, it is assumed that

$$\beta(\omega') = \beta_0 + A[\omega' - \omega_0] - B[\omega' - \omega_0]^2 \qquad (7)$$

in which

$$A = \frac{\omega_0}{v^2\beta_0}$$

$$B = \frac{1}{2}\frac{\omega_c^2}{v^4\beta_0^3}. \qquad (8)$$

$$F(t) = \frac{1}{2}\sqrt{\left\{\operatorname{erf}\left[\frac{x+1}{a}\right] - \operatorname{erf}\left[\frac{x-1}{a}\right]\right\}^2 + \left\{C\left[\left(\frac{x+1}{a}\right)^2\right] - S\left[\left(\frac{x+1}{a}\right)^2\right] - C\left[\left(\frac{x-1}{a}\right)^2\right] + S\left[\left(\frac{x-1}{a}\right)^2\right]\right\}^2} \qquad (14)$$

In a waveguide run of length L, the Fourier component at angular frequency ω' is shifted in phase an amount βL.

Assuming that the attenuation is frequency insensitive,[7] the output waveform is proportional to

$$f(t) = \frac{T}{2\pi}\int_{-\infty}^{\infty} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \cos\left[(\omega + \omega_0)t - \beta_0 L - AL\omega + BL\omega^2\right]d\omega. \qquad (9)$$

From (9) it follows that

$$f(t + AL) = \frac{T}{2\pi}\cos\left[\omega_0(t + AL) - \beta_0 L\right] \int_{-\infty}^{\infty} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \cos\left[\omega t + BL\omega^2\right]d\omega - \frac{T}{2\pi}\sin\left[\omega_0(t + AL) - \beta_0 L\right] \int_{-\infty}^{\infty} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \sin\left[\omega t + BL\omega^2\right]d\omega. \qquad (10)$$

The output modulation envelope is therefore given by

$$F(t) = \frac{T}{2\pi}\sqrt{\{F_1(t)\}^2 + \{F_2(t)\}^2} \qquad (11)$$

with

$$F_1(t) = \int_{-\infty}^{\infty} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \cos\left[\omega t + BL\omega^2\right]d\omega \qquad (12)$$

$$F_2(t) = \int_{-\infty}^{\infty} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \sin\left[\omega t + BL\omega^2\right]d\omega. \qquad (13)$$

If the trigonometric functions occurring in the integrands of (12) and (13) are replaced by their equivalent exponential forms, the Fourier transforms of (12) and (13) can be deduced with the repeated use of formula 731.1 of Campbell and Foster.[8] One concludes that (14)

with

$$x = \frac{2t}{T} \qquad (15)$$

$$a = \frac{4}{T}\sqrt{BL} \qquad (16)$$

and

$$\operatorname{erf}[z] = \frac{2}{\sqrt{\pi}}\int_0^z e^{-u^2}du \qquad (17)$$

$$C[z] = \sqrt{\frac{2}{\pi}}\int_0^{\sqrt{z}} \cos u^2 du \qquad (18)$$

$$S[z] = \sqrt{\frac{2}{\pi}}\int_0^{\sqrt{z}} \sin u^2 du. \qquad (19)$$

Erf$[z]$ is the error function and $C[z]$, $S[z]$ are the Fresnel integrals. Adequate tables of the error function are available for all values of z of interest in this analysis. Tables of $C[z]$, $S[z]$ are available for $0 \leq z \leq 50$. For $z > 50$, the asymptotic expressions, due to Cauchy,[9] may be used. The first two terms are sufficient, giving

$$C[z] - S[z] \cong \sqrt{\frac{2z}{\pi}}\left\{\left[\frac{1}{2z} + \frac{1}{(2z)^2}\right]\cos z + \left[\frac{1}{2z} - \frac{1}{(2z)^2}\right]\sin z\right\}. \qquad (20)$$

[7] This is obviously not true in the entire range $-\infty < \omega' < \infty$. However in the restricted range $\omega_0 - \Omega \leq \omega' \leq \omega_0 + \Omega$ needed to adequately represent the pulse, it is a reasonable assumption.

[8] G. A. Campbell and R. M. Foster, "Fourier Integrals for Practical Applications," D. Van Nostrand Co., Inc., New York, N. Y.; 1948.

[9] A. L. Cauchy, "Asymptotic expansions for Fresnel integrals," *C.R. Acad. Sci., Paris*, vol. 15, pp. 554, 573; 1842. See also, G. N. Watson, "Bessel Functions," Cambridge University Press, Cambridge, Eng., 2nd ed., p. 545; 1952.

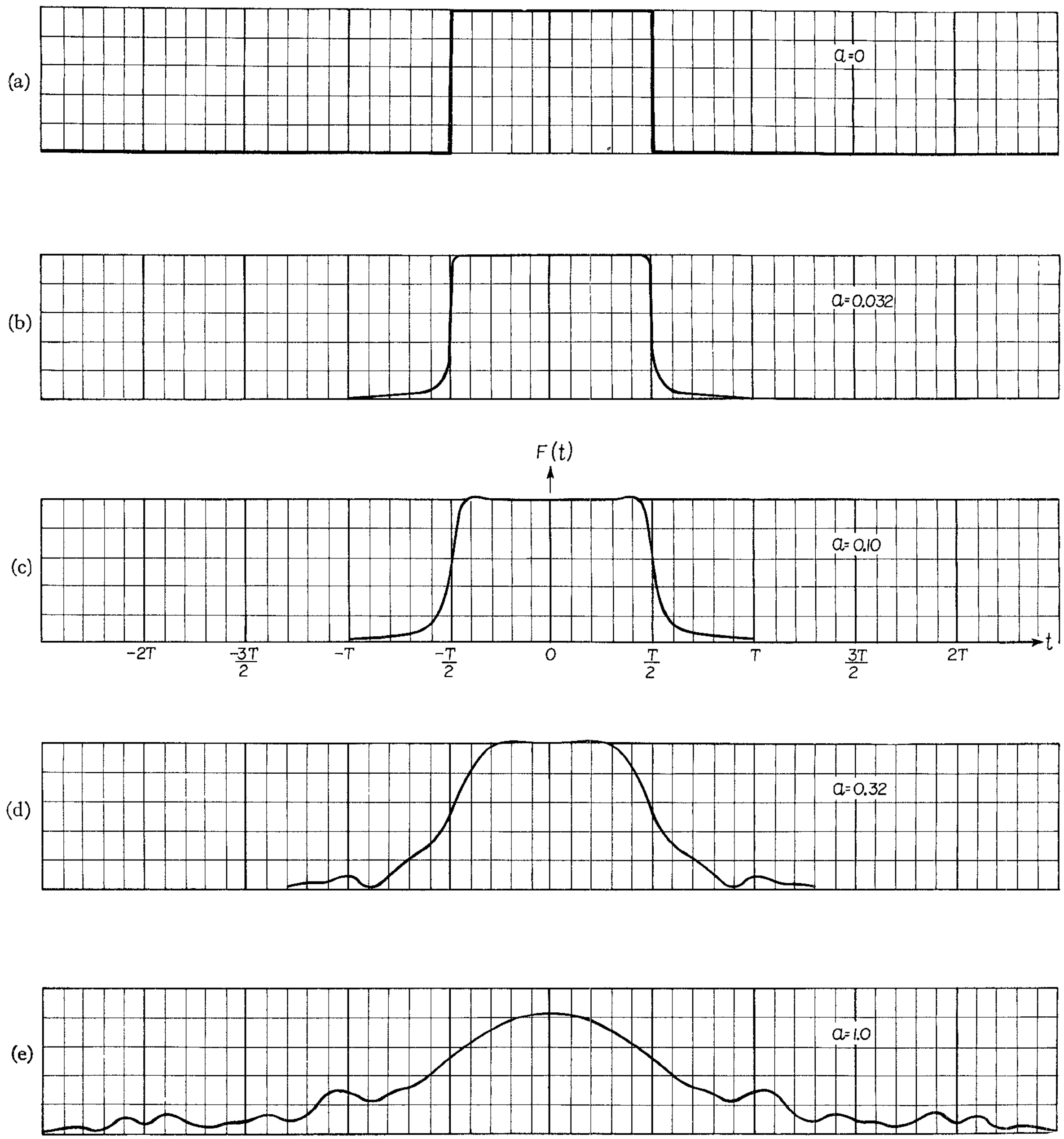

Fig. 2—Degraded waveforms.

Output waveforms have been computed from (14) for the parameter values $a=0$, 0.032, 0.1, 0.32, 1.0. These are shown in Fig. 2 and comprise a universal family of degraded waveforms. For any particular application, one need only compute B from (8) and a from (16). A rough estimate of the output waveform can be obtained from Fig. 2 by examining the curve for the appropriate value of a. A more exact estimate can be obtained by inserting the precise value of a in (14) and computing $F(t)$ with the aid of the tables of error function and Fresnel integrals.

A study of Fig. 2 reveals that the degradation increases with a. Thus, the narrower the pulse, the longer the waveguide run, the closer one operates to cutoff, the greater the pulse degradation. These are all reasonable results and (14) establishes the degree to which these factors have an influence.

As an example, consider a K_a band delay line consisting of circular waveguide 0.500 inch in outside diameter with a 0.032-inch wall, and carrying a TE_{01} mode. If the carrier frequency is 34,200 mc and the pulse width is 0.125 μsec, the round trip delay in a length of forty feet is only 0.3 μsec, and the exit waveform is approximately Fig. 2(c). By increasing the length to four hundred feet, the delay can be raised to 3 μsec, but the pulse is further degraded, being given approximately by Fig. 2(d).

Conclusion

By assuming that phase velocity in a waveguide is a quadratic function of frequency, it is possible to derive a compact formula for pulse degradation due to dispersion. From the formula, a family of universal curves can be computed, showing the effect of guide length, pulse width, and dispersion on the pulse shape.